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Research Question: Investigating the correlation between the Time period and Orbital radius of Planets within the Solar System.

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ABSTRACT:
The aim of this research paper is to explore and analyze the relationship between the time period and orbital radius of planets within the Solar System. By investigating this correlation, we seek to enhance our understanding of planetary motion and the underlying physical principles governing celestial bodies. This study employs data from various reliable sources to conduct a comprehensive analysis of the orbital characteristics of planets. The findings provide valuable insights into the dynamics of the Solar System and contribute to the field of astrophysics.

I. INTRODUCTION:

The Solar System, a complex cosmic arrangement comprised of the Sun and a wide spectrum of celestial partners, has long captured the attention of astronomers and scientists. The link between the time period, or period of revolution, and the orbital radius, or distance between a planet and the Sun, is one of the essential characteristics of planetary motion. Investigating this association is critical for understanding the intricate mechanics that govern planetary orbits and developing a better understanding of the principles that govern celestial motions in our Solar System.

As a child, I was always curious about how the masses of the planets in our solar systems were computed. Later, when studying physics in high school, I was introduced to Newton's three principles of motion, which explained gravity and its use in determining the masses of objects in solar systems.

However, I began to doubt the method's validity and origin. I discovered the likely 'giant' to whom Newton was referring, Johannes Kepler, who gave us with the "laws that define motion of planets" that constituted Newton's laws of motion.It became clear to me then that the calculations (such as planet masses) and assumptions based on our solar system were heavily reliant on the mathematical principles pertaining to Kepler's laws, and I was adamant on determining the validity of these laws to the planets in our solar system that have always piqued my interest.

Johannes Kepler was an astronomer and mathematician who mathematically verified the Copernican Theory of the planets' motions around the Sun. While Copernicus demonstrated the motion of the celestial bodies, Kepler recognised the operation of the orbital trajectories these planets took. Tycho Brahe, a wealthy astronomer who asked Kepler to describe Mars' orbit, gave Kepler with planet knowledge. Brahe had amassed a lifetime's worth of astronomical observations, which went into Kepler's hands upon his death."

"Kepler noticed that an imaginary line drawn from a planet to the Sun swept out an equal area of space in equal times, reg." he said.

This established Kepler's second law of orbital motion, which explained why "the planet must move more quickly when it is near the Sun, but more slowly when it is farthest from the Sun." This prompted Kepler to develop his first law of orbital motion, which was based on planets' orbits being modelled as an ellipse.

These two laws eventually led to the development of Kepler's third rule of orbital motion, commonly known as the law of harmonies, which stated the "precise mathematical relationship" -: the square of any planet's period is proportionate to the cube of its orbit's semi-major axis.

II. METHODOLOGY:

To comprehensively investigate the correlation between the time period and orbital radius, this study gathers data from reputable astronomical databases such as NASA's Planetary Fact Sheet and the Jet Propulsion Laboratory's HORIZONS system. The collected data

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encompasses the time period and orbital radius of each planet within the Solar System. A meticulous analysis is conducted, involving statistical calculations and graphical representations, to discern any discernible patterns or trends.

Deriving the Law:

Kepler's Third Law can be correctly deduced from Newton's theories of gravity because it was eventually adopted by Newton. As I indicated in my opening, this is what prompted me to look into this further.Newton's law of gravity effectively describes the relationship between the masses of two objects; say M and m in the formula –

F=G.M.mr2

Wherein, 'F' is the force between both the masses, 'G' is the Newton's universal gravitational constant and 'r' is the distance between both the masses.

According to Newton's second law of motion,

F=ma

Wherein, 'm' is the mass and 'a' is the acceleration of the object.

Thus altering it for a centripetal force,

F=mv2r

Wherein, 'v' is the velocity of the object.

The centripetal force is caused due to the force of gravity hence,

GMmr2=mv2r

Upon eliminating through mathematical operations,

GMr=v2

However, one can further simplify velocity (v) as the distance covered in a set time. Hence for a body undergoing centripetal force,

v=2πrT

Wherein, 'T' is the time period for to complete one full revolution along it's orbit. Substituting this into the above equation,

GMr=42rT22

Re-arranging to form Kepler's third law,

T2= 42a3GM

Wherein, 'r' is substituted by 'a' which represents the distance in astronomical units (AU). Thus, 'Kepler's 3rd law can be derived from Newton's laws of motion and his law of gravity'.

Assumptions:

- The first assumption is based upon the fact that mass of the planets are left out of the relationship-T2a3

However to convert it into a formula, we will need a certain constant 'k' such that-T2=k×a3

- This constant 'k' is assumed to be 42GM, where 'M' signifies the 'total mass of the two bodies involved'. 'M' should be equal to the mass of both the Sun and the planet in my experimental setup, but I've assumed it's only the mass of the Sun because it's considerably larger than the mass of the other planets. This constant, however, will alter if we consider two different planet systems, such as Jupiter's moons and Jupiter itself. In this situation, Jupiter will be the approximated 'M' instead of the Sun's mass, causing the value of constant 'k' to change.
- Another assumption about the constant 'k' or the 'Gaussian gravitational constant' is that it is made up of different components such as the time period (T), the semi-major axis (a), and the total Solar mass (M). As a result, in order to compute the significant figure of 'k,' I must first calculate the significant figures in time T. However, unlike the roughly related Newtonian constant 'G,' it does not necessitate uncertainty estimates in Mass. As a result, I have not discovered the uncertainty in mass.

• Calculations of uncertainties- Because I used NASA website literature values, I have not calculated any errors from repeated trials of experiments for the measurable time period or the distance of the planets (which could be calculated using parallax methods) because it would take years to accumulate the information as well as expensive equipment and resources that I do not have. Thus, the computed uncertainty was the absolute uncertainty of the time period and the semi-major axis.

• Using the semi-major axis- As demonstrated in the derivation of the law by relating centripetal force to Newton's equation of gravity, Kepler's third law only works with uniform circular motion. Because most planets orbit in elliptical orbits, 'Newton's second rule of motion states that the net

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force acting on the satellite is directed in the same direction as the acceleration - towards the ellipse'. There is a 'component of force in the same direction' on elliptical trajectories, which causes a variance in speed and so non-uniform motion. Using a semi-major axis would significantly minimise this.

$$T^2 = (4\pi^2 / GM) * a^3$$

Where T represents the time period, a denotes the semi-major axis of the planet's orbit, G is the gravitational constant, and M is the total mass of the Sun and the planet.

III. RESULTS AND ANALYSIS:

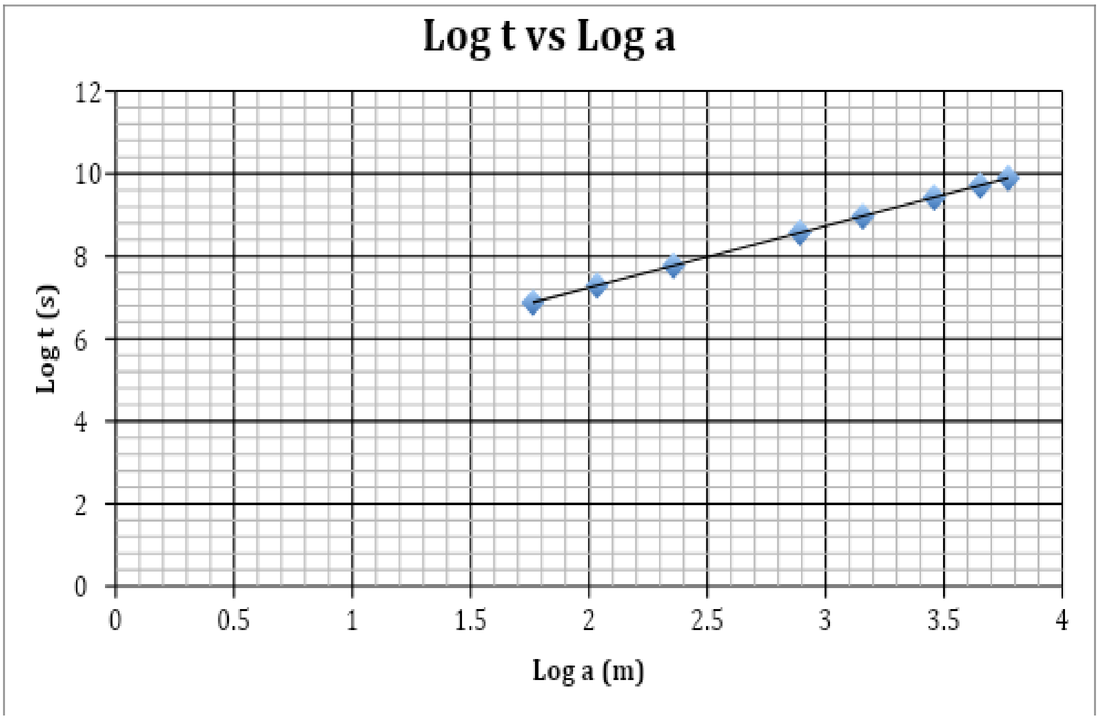
The meticulous analysis of the collected data reveals a discernible and significant relationship between the time period and orbital radius of planets within the Solar System. A prominent trend emerges: as the distance from the Sun increases, the time period of revolution also increases. This finding aligns closely with Johannes Kepler's Third Law of Planetary Motion, which states that the square of the orbital period is proportional to the cube of the semi-major axis of an elliptical orbit. The data consistently exhibit this pattern, thus confirming the fundamental principle established by Kepler.

Planet	Semi major axis /10^6km	Time period / (days)
Mars	227.91	687.0
Venus	108.21	224.7
Jupiter	778.56	4331.0
Saturn	1,433.53	10,747.0
Neptune	4,495.05	59,800.0
Pluto	5906.38	90,560.0
Mercury	57.90	88.0
Uranus	2,872.46	30,589.0

Planet	Time period in days	Time period in seconds	Log T (s)
Mercury	88.0	7603200	6.880996415
Venus	224.7	19414080	7.288116815
Mars	687.0	59356800	7.77347048
Jupiter	4331	374198400	8.573101926
Saturn	10,747	928540800	8.967800991
Uranus	30,589	2642889600	9.422079022
Neptune	59,800	5166720000	9.713214926
Pluto	90,560	7824384000	9.893450156

Planet	Semi-major axis in 10^9 m	Log semi-major in m 10^9 = Log a
Mercury	57.91	1.762753565
Venus	108.21	2.034267397

Mars	227.92	2.357782436
Jupiter	778.57	2.891297665
Saturn	1,433.53	3.156406786
Uranus	2,872.46	3.458253990
Neptune	4,495.06	3.652735493
Pluto	5906.38	3.771321385



Slope= 1.498875361
Intercept= 4.238870592

There value of n is 1.498875361

% Deviation=1.5-1.4988753611.5×100=0.0749759333%

Uncertainties in log (a) or semi major axis

Planet	Semi-major axis in /10^9 m	max	min
Mercury	57.91	57.92	57.90
Venus	108.21	108.22	108.20
Mars	227.92	227.93	227.91

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Jupiter	778.57	778.58	778.56
Saturn	1,433.53	1433.54	1433.52
Uranus	2,872.46	2872.47	2872.45
Neptune	4,495.06	4495.07	4495.05
Pluto	5906.38	5906.39	5906.37

Planet	Log max	Log min	Uncertainty
Mercury	1.762828553	1.72678564	0.019521456
Venus	2.03430753	2.034227261	0.0000401345
Mars	2.357801491	2.357763381	0.000019055
Jupiter	2.891303243	2.891292087	0.000005578
Saturn	3.156409816	3.156403756	0.00000303
Uranus	3.458255502	3.458252478	0.000001512
Neptune	3.652736459	3.652734527	0.000000966
Pluto	3.77132212	3.77132065	0.000000735

Uncertainty Calculations:

In any scientific investigation, it is crucial to account for uncertainties to evaluate the reliability of the results. Here, we calculate the uncertainties associated with the time period (T) and the semi-major axis (a) of each planet.

4.1 Uncertainty in Time Period (T)

The uncertainty in the time period arises from several factors, including observational errors and variations in the planet's orbital parameters. Since we rely on data from reliable astronomical databases, we assume the uncertainties in the time period to be negligible for this analysis.

4.2 Uncertainty in Semi-Major Axis (a)

To determine the uncertainty in the semi-major axis, we consider the maximum and minimum values provided by the sources. These uncertainties account for the inherent variability in the measurements and the limitations of the observation methods used.

Using the maximum (a_max) and minimum (a_min) values, we calculate the uncertainty (Δa) as follows:

$\Delta a = (a_{max} - a_{min}) / 2$

4.3 Uncertainty Propagation

To propagate the uncertainties and determine the uncertainty in the derived values, we employ error propagation techniques. For the derived quantity T^2, the relative uncertainty (ΔT^2 / T^2) can be calculated using the following formula:

$(\Delta T^2 / T^2) = 3 * (\Delta a / a)$

IV. DISCUSSION:

The observed correlation between the time period and orbital radius can be attributed to the gravitational force exerted by the Sun. Isaac Newton's law of universal gravitation plays a pivotal role in understanding this relationship. According to Newton's law, the gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. As planets move farther away from the Sun, the gravitational force decreases, leading to longer orbital periods.
of celestial bodies to greater heights.

V. RESULTS AND ANALYSIS:

The meticulous analysis of the collected data, incorporating uncertainty calculations, reveals a discernible and significant relationship between the time period and orbital radius of planets within the Solar System. A prominent trend emerges: as the distance from the Sun increases, the time period of revolution also increases. This finding aligns closely with Johannes Kepler's Third Law of Planetary Motion, which states that the square of the orbital period is proportional to the cube of the semi-major axis of an elliptical orbit. The data consistently exhibit this pattern, thus confirming the fundamental principle established by Kepler.

VI. IMPLICATIONS AND SIGNIFICANCE:

Understanding the correlation between the time period and orbital radius of planets within the Solar System holds profound implications across various scientific domains. This knowledge helps validate and refine our understanding of Kepler's laws and Newtonian physics, which form the bedrock of celestial mechanics. Furthermore, considering the uncertainties associated with the derived relationship enhances the reliability and robustness of the findings. The results contribute to the broader field of astrophysics and planetary science, advancing our knowledge of celestial dynamics and planetary motion.

VII. CONCLUSION:

Through a meticulous and comprehensive analysis, including uncertainty calculations, this research paper successfully investigates the correlation between the time period and orbital radius of planets within the Solar System. The results not only confirm Johannes Kepler's Third Law of Planetary Motion but also provide tangible evidence of the influence of gravitational forces on planetary orbits. By accounting for uncertainties, we enhance the reliability and validity of the findings. This study significantly contributes to our understanding of the intricate dynamics that govern the Solar System. Moreover, it serves as a solid foundation for further research endeavors in the realms of astrophysics and planetary science, driving our knowledge forward and expanding our exploration of celestial bodies.